701. Consider the range of the functions $x \mapsto \cos x$ and $x \mapsto 2^x$.

702. The relevant fact is that for a large population with distribution $X \sim N(\mu, \sigma^2)$, the means of samples of size n are distributed as

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right).$$

703. (a) Differentiate and substitute in.
   (b) "The best linear approximation at $x = 2$" is the tangent line at $x = 2$.

704. This is a quadratic in $\cos x$: factorise.

705. Consider possible vertical stretches.

706. For the average speed, calculate the displacement over the five second period. For the instantaneous speed, differentiate.

707. Substitute for x or y, and you'll get a quadratic. Find an algebraic factorisation to determine the solution.

708. Look for a counterexample.

709. Translate the first sentence into an equation, and differentiate it.

710. Find the average of the angles, by considering the sum of the interior angles of a polygon. Since the angles are in AP, the average of the smallest and largest must equal the average of all five.

711. (a) Substitute in for x, remembering that $a^0 = 1$.
   (b) Multiply both sides by a.

712. (a) Subtract the area of three right-angled triangles from the area of the square.
   (b) Differentiate to find $\frac{dA}{dk}$.
   (c) Set $\frac{dA}{dk} = 0$.

713. Complete the square or differentiate to find the stationary value of each. Mention also the fact that the leading coefficient is positive in both.

714. Solve pairs of boundary equations to find the three intersections of the boundary lines. Then show that the triangular region which satisfies all three inequalities lies inside one integer grid square.

715. (a) Use u_1 to generate u_2, then u_2 to generate u_3, and so on.
   (b) Use $u_1 = 1$ (i.e. set $n = 1$) to find the value of b. Then use $u_2 = 2$ to find the value of a.

716. The statement is true. Think in terms of (signed) areas on a graph.

—— Alternative Method ——

Set up the indefinite integral F with $F'(x) = f(x)$ and proceed algebraically.

717. Rearrange $v = u + at$ to make t the subject. Sub this into $s = ut + \frac{1}{2}at^2$ and rearrange.

718. You might find sketching a number line helpful.

719. The line $x = k$ is parallel to y. The normal to a curve is only parallel to y at stationary points.

720. Set $x = 0.\dot{2}3\dot{7}$, then calculate $1000x - x$.

721. Use $F_{\text{grav}} = -mg$; set up and carry out the definite integral of this force between $x = h$ and $x = 0$.

   Alternatively, use $F_{\text{grav}} = mg$; set up and carry out the definite integral of this force between $x = 0$ and $x = h$.

722. Consider the fact that $x = 2$ is a stationary point of $y = f(x)$, and classify the stationary point with the second derivative.

723. Show that both ratios are $1 : \sqrt{2}$.

724. Look for counterexamples: pairs of values (x, y) that satisfy the "if" but not the "then". The set $\mathbb{R} \setminus \mathbb{Q}$ is the set of reals minus the set of rationals, i.e. it is the set of irrationals.

725. (a) Show that ${}^n\text{C}_1$ is equal to n.
   (b) Simplify the numerator. Then divide top and bottom by h. At this point you can take the limit (i.e. send h all the way to zero), as there is no division by h.

726. No calculations are needed here: just consider the symmetry of the possibility space.

727. (a) Find the first and second derivatives $h'(x)$ and $h''(x)$, with coefficients in terms of a, b and c. Then substitute in the relevant values.
   (b) Factorise the cubic: you should find a single and a double root.

728. The justifications for parts (a) and (b) are two of Newton's laws. Each answer is one law.

729. For the vertical asymptote, look for the root of the denominator. Then, for the horizontal asymptote, consider the behaviour as $x \to \infty$. Analyse this by first dividing top and bottom by x.

730. The terms have a common factor of $\sqrt{2}$; simplify the surds and take it out. Then complete the square in the quadratic factor, before multiplying the $\sqrt{2}$ back in.

731. (a) This is the mean (signed) deviation from the mean. By definition, this always has the same value, regardless of the sample involved.
(b) For a sample element x_i, the quantity $(x_i - 10)^2$ is the squared deviation from the mean (in this case 10). The variance is defined as the mean squared deviation from the mean.

732. Differentiate (i.e. apply the differential operator $\frac{d}{dx}$) term by term. The question says "Simplify", so you won't get a numerical value or an explicit formula here; your answer will involve $\frac{dy}{dx}$.

733. Consider whether there are any real values of x which may not appear in the domain i.e. the set of inputs of each function.

734. Scales measure contact reaction force. If this is overestimated by 20%, then the reaction force is $1.2mg$ on an object of weight mg. Draw a force diagram and use NII to find the acceleration.

735. One is true, the other false.

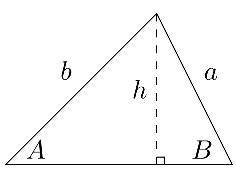
736. Complete the square for x and for y to determine the centre and radius of the circle. Then use the fact that tangent and radius are perpendicular.

737. This is a GP. Set up a boundary equation using the standard ordinal formula $u_n = ar^{n-1}$, and solve using logarithms. Remember that $n \in \mathbb{N}$.

738. Express each set as an interval, before combining. The answer is an interval. Square brackets include the corners, while round brackets don't.

739. (a) The length is that of two straight sections and the circumference of one whole circle.
(b) i. Consider symmetry.
ii. Consider the tension in the band.
(c) Draw a force diagram for one cane in isolation, with two tension forces acting on it.

740. Drop a perpendicular as follows:



Calculate the height h in two different ways.

741. Substitute the s values at the endpoints, and use Pythagoras.

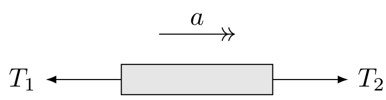
742. Use the factor theorem, noting that $(-6, 0)$ and $(3, 0)$ are x intercepts.

743. Consider the cards one by one, multiplying the probabilities of success as you go. The cards are being dealt with replacement, so the numerators and denominators do not change.

744. (a) Consider the sets defined with modulus signs as intervals.
(b) Moving on from part (a), consider whether changing the x's for y's makes any difference.

745. This question is not about the discriminant. The *quadratic* has roots, but the individual factors do not. Factorise and consider the range of the cosine function.

746. Consider NII for the string, whose mass is modelled as negligible:



747. Consider the distance between the centres of the two circles, compared to their radii.

748. The left-hand expression is not well defined.

749. Count the vertices, edges and faces.

750. Start by setting up the equation for intersections. Show that its discriminant is zero.

751. "Nano" is $\times 10^{-9}$.

752. (a) Rearrange to $16p^2q^2 - 1 = 0$.
(b) Rule out the negative value of pq, and use the positive value to solve simultaneously.

753. Consider the minimum value of the squared term.

754. (a) The relevant values are $t = 0$ and $t = 1$.
(b) Having set up a vector equation in this way, with $t = 0$ at B and $t = 1$ at A, the parameter t acts like a scale running linearly between A and B. To divide AB in the ratio $1 : 3$, use the parameter value $t = 3/4$.

755. Set up a general first derivative $f'(x) = ax + b$, and differentiate it to show that the second derivative is necessarily constant.

756. Since $a, b \neq 0$, the equation is quadratic. So, if it has precisely one root, then

    ① its discriminant must be zero,

    ② it must have a repeated factor.

    You can use either of the equivalent facts above to show that $a^2 = b^2$.

757. (a) $(p - q)$ and $(q - p)$ are negatives.

    (b) Use a difference of two squares, and the same fact as in part (a).

758. Use the cosine rule to find $\cos\theta$, where θ is one of the internal angles. Then use the first Pythagorean identity to find $\sin\theta$, and use the sine area formula.

    Alternatively, you could, if you know it, use Heron's formula.

759. Start with a cuboid, side lengths (a, b, c).

760. Write the sum out longhand (i.e. not using sigma notation), and find the terms individually. Each is a standard exact value.

761. (a) Write down the answer, considering pirate and parrot as one system.

    (b) Draw a force diagram for the parrot alone. This diagram should contain the Newton III pair of the force in question.

762. A root of an equation $\text{f}(x) = a$ is an element x (represented by a dot) in the set D, which maps to a. So, pick an element in C and ask "How many elements in D map here under the action of f?"

763. This is a quadratic in $\sin x$. Solve by factorising. There are three roots in $[0, 360°)$.

764. Set up the equation for intersections. Factorise or use the discriminant.

765. The *expected* percentage change is the *average* percentage change under such a procedure. For purposes of calculation, you can assume that the twenty data selected are evenly spread through the sample. In other words, 20% of the sum $\sum x$ is scaled by 0.75 and 80% is not scaled. These can be combined into a single scale-factor calculation.

766. Use the second fact to show that the right angles must be opposite one another. Then use a circle theorem.

767. The equation for fixed points is $\text{f}(\alpha) = \alpha$. Apply the inverse of f to both sides.

768. Draw a force diagram for each, using $F_{\text{max}} = \mu R$. Note that this formula only gives the *maximum possible* magnitude of the frictional force, not the *actual* magnitude of the frictional force.

769. The new parabola must be monic i.e. must have leading coefficient 1. Consider the effect of the transformation on the x intercepts, and use the factor theorem.

770. Opposite faces of a die add up to seven. Show that the vertical faces of the dice contribute 3×14 dots, and then consider the uppermost face.

771. (a) These events sum to 1.

    (b) If A and B are independent, then so are A' and B'. The definition of independence for A' and B' is $\mathbb{P}(A' \cap B') = \mathbb{P}(A') \times \mathbb{P}(B')$.

772. In each case, use the word "negligible".

773. Express 2 as 16^k. Then use $\left(a^b\right)^c \equiv \left(a^c\right)^b$.

774. Two simultaneous equations in two unknowns are solved by reducing them to one equation in one unknown. Likewise, three simultaneous equations in three unknowns are reduced to two in two, then further reduced to one in one, and then solved.

775. Consider an equilateral triangle of side length 2. Drop a perpendicular and use Pythagoras.

776. Consider how it is possible for two lines to intersect at infinitely many points.

777. Sketch the lines. Note that the distance between parallel lines means the *shortest* distance between them, which is normal to both.

778. (a) i. Consider NIII, given that the cable is light.

        ii. Consider NII.

    (b) Find the acceleration of the astronaut *relative to* the spacecraft. Use this to find the position and velocity of the astronaut, relative to the spacecraft, at the end of the five seconds. Note that, after the winch has cut out, the velocity remains constant.

779. The information given does not allow you to find f explicitly. However, you don't need to do so. This is because 0 is the midpoint of the interval $[-2, 2]$, so a symmetry argument applies. Let $\text{f}(x) = ax + b$ and carry out the integral to find b.

780. The key is whether each statistic is a measure of central tendency or of spread. Spread is unaffected by negation.

781. Write the formula relating the derivatives as an equation with constant of proportionality k. Then integrate it with respect to z, remembering the constant of integration.

782. Test the quantity $x^4 + y^4$.

783. The argument gives a correct result, but there is a piece of logical justification missing, regarding division by 2^{2x}.

784. Scales measure contact force, specifically reaction force. If mass is overestimated by $k\%$, then the scales are registering a force of $(100 + k)\%$ of mg.

785. By substituting $s = 0, 8$ and $t = -4, 4$, find the coordinates of the endpoints of the line segments.

786. Evaluate the integral with and without the small-angle approximation. Since you've only got to *find* the answer rather than *determine* it, you don't need to do this analytically using algebra: use the definite integration facility on a calculator.

787. Take out a common factor first, noting that this common factor will produce a root.

788. Consider $x = 0.5, 1.5, 2.5, 3.5$. Each of these is a counterexample to one of the implications.

789. Use the factor theorem.

790. Use the fact that there is a double root at $x = 2$, so the cubic has a factor of $(x - 2)^2$.

791. Assume, for a contradiction, that there are real numbers $a, b \neq 0$ such that $a\mathbf{p} + b\mathbf{q} = 0$. Use this to show that $\mathbf{p}$ and $\mathbf{q}$ must be parallel (which is a contradiction).

792. Remember to include the constants of integration. You need a different algebraic symbol each time you integrate.

793. Rewrite the second equation with v as the subject. Substitute this into the first equation.

794. Substitute in the values $x = 0$ and $x = 1$; this gives you values for A and B. Then equate the coefficients of x^4. Alternatively, you can multiply both sides out and compare coefficients directly.

795. Scale each test to be out of 50 marks. Then add the two scores.

796. (a) Factorise.
   (b) Start with the second of the results from part (a). Rearrange algebraically to the required result, dividing through by two to produce the relevant mean.

797. Sketch the curves. Each is a transformed version of the reciprocal graph $y = 1/x$: both have been translated in the x direction.

798. A general odd number is $2a + 1$, for $a \in \mathbb{Z}$.

799. Consider picking the cards one by one. Compare the number of successful outcomes remaining for the second card.

800. One of the results holds. Consider $a = b \neq c$ as a counterexample.

——— End of 8th Hundred ———